Observations of a liquid-into-liquid jet

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A jet of dyed water is directed into a large tank of water. As the flow rate is increased several successive modes of breakdown of the dyed column are observed:
(*a*) tiny puffs of dye near the nozzle;
(*b*) axisymmetric condensations well away from the nozzle;
(*c*) sinuous undulations of long wave length far from the nozzle;
(*d*) formation of foot-shaped pockets of dye;
(*e*) confused breakup near the nozzle with rapid spread of dye.
The length of nearly rectilinear jet increases through the stages (*a*), (*b*) and (*c*). The mode (*d*) is an alternative to (*c*), occurring nearer the nozzle and leaving a shorter length of straight jet. Above a Reynolds number of about 300, only the confused breakup (*e*) is found.

This experimental investigation was provoked by two widely differing estimates of the critical Reynolds number for an axisymmetric jet through a liquid of the same density. These estimates were based on experiments done several years ago in Cambridge, England, by H. Schade and more recently in Cambridge, Massachusetts, by A. Viilu. According to Dr G. K. Batchelor, Schade found that a long, nearly rectilinear jet could be maintained until the Reynolds number was a few hundred. Viilu (1960) reported, after careful experiments at low flow rates, the value $11{\cdot}2 \pm 0{\cdot}7$ for the Reynolds number of breakdown.

Here and subsequently we take the Reynolds number characterizing a jet to be

$$R = \frac{Vd}{\nu} = \frac{4Q}{\pi \nu d},$$

with V the mean velocity through the nozzle producing the jet,
d the diameter of the nozzle,
ν the kinematic viscosity of the liquid, and
Q the volume flow through the nozzle.

In the present tests the range of the Reynolds number above Viilu's critical value is investigated in order to discover what are the phenomena that led Schade to consider a jet stable there. Only a general survey has been attempted; the refinements necessary for a definitive quantitative study were not made to the apparatus.

Details of tests

A jet of dyed water was directed into a tank of water through a glass tube drawn down to form a nozzle (nozzle diameter $0{\cdot}32 \pm 0{\cdot}05$ mm). The dye used was Eosine, in concentrations so small that the properties of the jet were not altered greatly from those of the ambient water. The tank was a glass cylinder 120 cm high and 30 cm in diameter. The injection tube entered through the free surface at the top of the tank, the nozzle being directed downwards along the axis. A constant-head device allowed sufficiently long test periods, typically about 20 min. Dye flow rates were determined by weighing the dye and reservoir before and after each run.

Before dye was injected, the water was allowed to stand in the tank overnight. However, as the apparatus was not located on a well-supported floor, small impulses were continually transmitted to it during the tests. Further, the nozzle used was noticeably asymmetric. Thus the experimental conditions were not unrealistically controlled; the phenomena reported may be expected to have counterparts in practically-occurring situations.

A strongly cautionary note must be introduced here. It may appear plausible that the effects of the tank walls on a jet from such a tiny nozzle are negligible. And this impression is reinforced by the narrowness of the observed dye column. But the diffusivities of momentum (the kinematic viscosity) and of the dye differ by an enormous factor; the momentum jet will spread very much more rapidly than the dye jet. This must be kept in mind not only in considering wall interactions, but also in interpreting the motions of the dye filament in terms of the momentum jet of which it forms merely the central part.

For nearly rectilinear flow at moderately large Reynolds numbers the laminar boundary-layer equations provide a useful approximation and the width of the momentum jet can be estimated using Schlichting's (1955) similarity solution for an axisymmetric jet. We find that

$$r_{\frac{1}{4}} = \left(\frac{64\pi\nu\mu}{3M}\right)^{\frac{1}{2}} x, \tag{1}$$

with $r_{\frac{1}{4}}$ the radius at which the axial velocity component drops to a quarter of it peak value,

x the axial distance from a virtual origin, and

M the axial momentum flux of the jet.

Assuming the jet momentum flux equal to that of a Poiseuille flow through the generating nozzle, we find that

$$r_{\frac{1}{4}} = \frac{8}{R} x.$$

Observations

A surprising variety of flows was encountered: nearly parallel jets of various lengths, sometimes extending to the bottom of the tank; disturbances in the form of symmetric condensations or, on other occasions, asymmetric forms growing out of the main column to form foot-shaped concentrations of dye; early instabilities near the nozzle which soon disappeared to leave a parallel column; and sometimes a rapid breakdown into a confused, rapidly spreading jet. An attempt has been made to reduce the observations to an assimilable form by plotting the lengths of simple (that is, nearly parallel) jet against the corresponding flow rates through the nozzle (figure 1). It will be noted that the variation of jet length departs widely from the monotonically decreasing function of

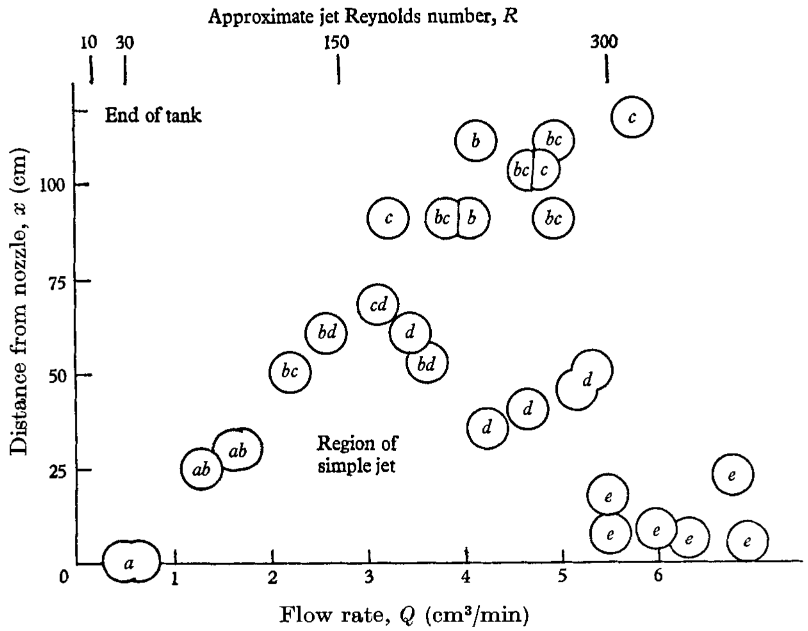

FIGURE 1. Positions of breakdown of simple jet plotted against flow rate, Q. In table 1 the symbols a, b, c, and d are related to modes of breakdown.

(*a*) *Shearing puffs*. These were found in the first few inches of a jet, often giving way to a steady column farther from the nozzle. They are possibly associated with transition from nozzle to jet velocity profile. These disturbances are sketched in figure 2. The saw-tooth form is produced by the shear of the momentum jet which extends well beyond the dye column.

(*b*) *Symmetric condensations*. These formed spontaneously well away from the nozzle, being usually from 3 to 5 cm apart, with visible diameter 5–10 mm, that is, two to four times the width of the basic column of dye. Often a steady train of nearly identical condensations was formed. But under some conditions, when not evenly spaced, they passed through one another in the manner of smoke rings. Similar condensations could sometimes be formed near the nozzle by giving the apparatus a light blow.

(*c*) *Sinuous undulations*. Those denoted by (*c*) in figure 1 were of wavelength about 25 cm and occurred in the lower portions of 'fairly stable' jets in which the width of the dye column was about 3 mm. However, short wavelength (2–3 cm) undulations often occurred in the first foot of a jet, especially just after a relatively 'stable' situation had been subjected to a slight shock. The width of the coloured region of the jet was then about 1·5 mm.

(*d*) *Pedal breakdown*. The development of the foot-shaped disturbances is sketched in figure 3. After this kind of breakdown, the visible jet was in the form of pockets of high dye concentration. Typically, these blobs (about 2 cm in diameter and about 1 cm thick) were spread through a cylinder some 15 cm in diameter and hence were well separated by fluid free from visible contamination.

(*e*) *Confused breakdown*. At the highest flow rates the rectilinear jet broke down abruptly within a few inches of the nozzle into a confused and rapidly spreading jet. At lower flow rates a shock could give rise to confusion near the nozzle. But then the confused region was usually convected steadily down the jet, which was otherwise undisturbed by the shock.

TABLE 1. Forms of breakdown of simple jet

flow rate that might have been expected, had we not been aware of Schade's observations. The rapid breakdown at low Reynolds numbers (that studied by Viilu) gives way to a progressively longer simple jet, which ultimately stretches the length of the tank, some 3800 nozzle diameters. However, a slight further increase in flow rate sharply reduces the length of the steady jet to a few inches. The Reynolds number at which this abrupt transition takes place is found to be $R \simeq 300$ on taking $d = 0{\cdot}32\,\text{mm}$, $Q = 5{\cdot}5\,\text{cm}^3/\text{min}$, $\nu = 0{\cdot}012\,\text{cm}^2/\text{sec}$.

FIGURE 2. Form of breakdown near the nozzle at low flow rates: the shearing puffs, mode (*a*) of table 1.

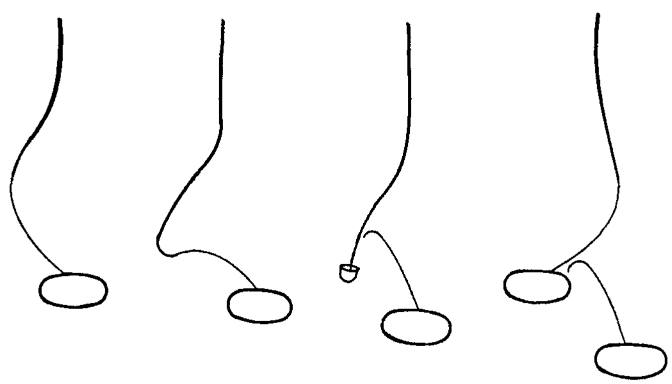

FIGURE 3. Successive stages of the pedal breakdown, mode (*d*) of table 1.

A unique correlation between the length of simple jet and the flow rate cannot be expected, since the choice of the point at which the dye column becomes non-uniform is not an objective one. Indeed, this point varies from moment to moment. Moreover, no account has been taken of day-to-day variations in viscosity. Nevertheless, the circles representing the observations are grouped fairly systematically in figure 1. The pattern becomes clearer when reference is made to the letters enclosed by the circles. These symbols indicate the kind of flow which supersedes the steady jet in each case, according to the code set forth in table 1. Over a lower range of flow rate ($Q < 3\,\text{cm}^3/\text{min}$) the experimental points define a single curve. But for higher flows ($3 < Q < 5{\cdot}5\,\text{cm}^3/\text{min}$) the observations separate into two groups. One of these is associated with the sinuous breakdown (*c*) of table 1 and continues upwards the curve found at lower flow rates. The other group, marked by the 'pedal' breakdown ((*d*) of table 1; see also figure 3), corresponds to shorter lengths of simple jet.

To see whether the walls are likely to affect the break-up of the straight jet, we can estimate the width of the momentum jet using the relation (1). The value of x/R for the higher branch in figure 1 is nearly constant at about 0·42 cm. Then $r_{\frac{1}{4}} \simeq 3{\cdot}5$ cm. Since the tank radius is 15 cm, it seems unlikely that the walls influence the initial breakdown.

Conclusions

These observations explain the discrepancy between the results of Schade and Viilu. While it is difficult to obtain a long steady jet at relatively low Reynolds numbers ($10 < R < 30$), progressively longer jets can be maintained as R increases towards 150. For higher Reynolds numbers ($150 < R < 300$) still longer simple jets can exist. But a complex breakdown (the pedal instability of figure 3) can also occur, with a reduction in length of simple jet as R increases towards 300. For $R > 300$ jets become disordered near the nozzle, although not so near as in the range $10 < R < 30$.

The ranges defined here are marked in figure 1. In view of the limitations of the experiments, the values of the bounding Reynolds numbers cannot be accepted as final.

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